SU(2) symmetry: three generators
SU(3) symmetry: eight generators

The generators do not commute! $\Rightarrow$ they correspond to a set of incompatible observables

For SU(2), therefore, we define the states in terms of the third component of the isospin, I_3, and the total isospin I

$$\hat{I}^2 = \hat{I}_1^2 + \hat{I}_2^2 + \hat{I}_3^2$$

In SU(3) the analogue of the total isospin is

$$T^2 = \sum_{i=1}^{8} \hat{T}_i^2 = \frac{1}{4} \sum_{i=1}^{8} \lambda_i^2 = \frac{4}{3} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

In SU(3) only two of the eight generators commute

$$\hat{T}_3 = \frac{1}{2} \lambda_3 \quad \text{and} \quad \hat{Y} = \frac{1}{\sqrt{3}} \lambda_8$$

third component of isospin   hypercharge: $Y = 2Q - 2I_3$

$$I_3 = n_u - n_d \,, \quad Y = \frac{1}{3}(n_u + n_d - 2n_s)$$

For antiquarks I_3 and Y have opposite signs compared to the quarks and form a $\bar{3}$ multiplet

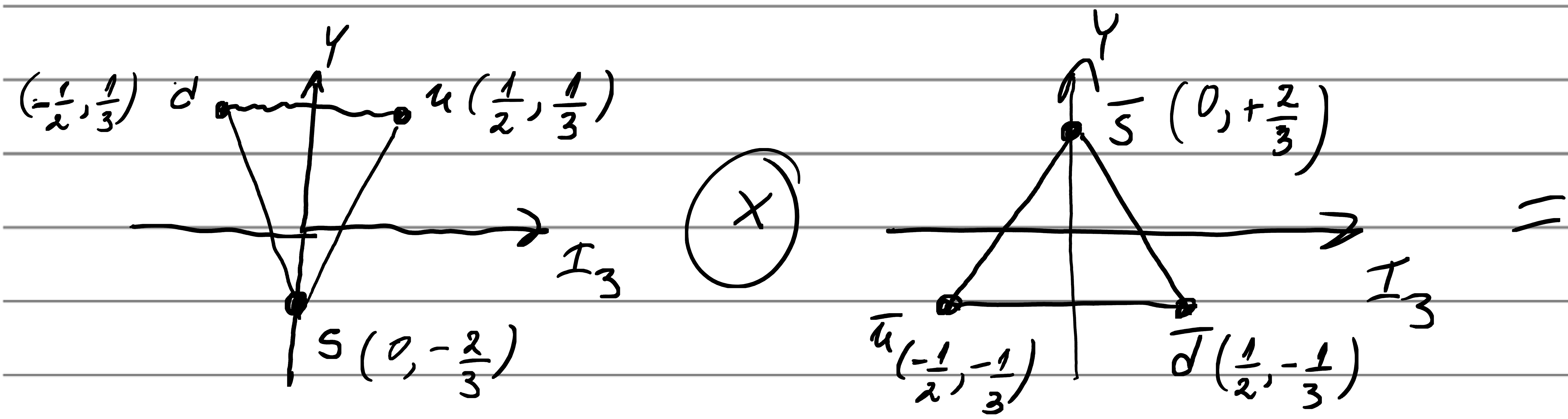
λ_3 and λ_8 label the SU(3) states, the rest of the generators are used to construct the ladder operators

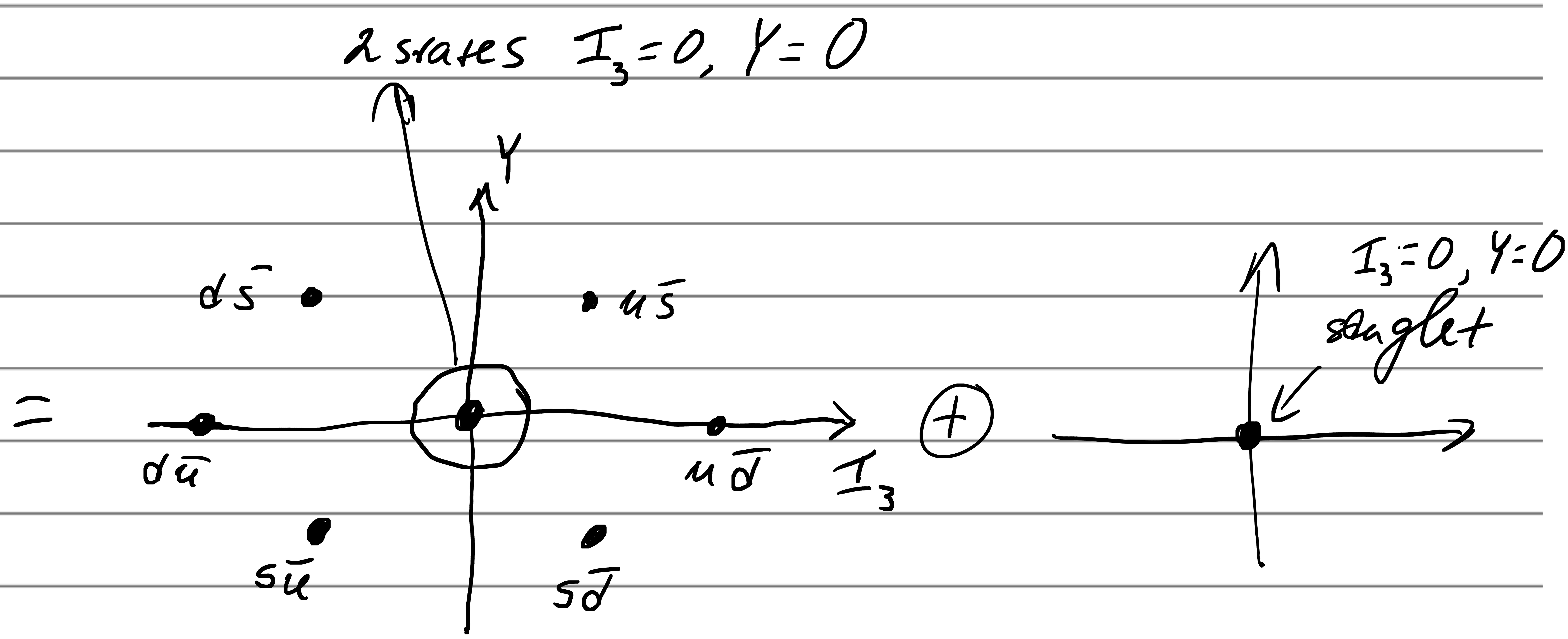
$$\hat{T}_\pm = \frac{1}{2}(\lambda_1 \pm \lambda_2) \qquad \hat{V}_\pm = \frac{1}{2}(\lambda_4 \pm \lambda_5) \qquad \hat{U}_\pm = \frac{1}{2}(\lambda_6 \pm \lambda_7)$$

$$\hat{V}_+ s = +u \quad \hat{V}_- u = +s \quad \hat{U}_+ s = +d \quad \hat{U}_- d = +s \quad \hat{T}_+ d = u \quad \hat{T}_- u = d$$

$$\hat{V}_+ \bar{u} = -\bar{s} \quad \hat{V}_- \bar{s} = -\bar{u} \quad \hat{U}_+ \bar{d} = -\bar{s} \quad \hat{U}_- \bar{s} = -\bar{d} \quad \hat{T}_+ \bar{u} = -\bar{d} \quad \hat{T}_- \bar{d} = -\bar{u}$$

The extreme states can be identified as the edges of a triangle in the (I_3, Y) plane:





$$3 \times \bar{3} = 8 \oplus 1$$

Meson masses

How can we explain the mass difference between the pseudoscalar and vector mesons?

- the strange quark is more massive than the u and d quarks
- however the vector mesons ($l=1$) have the same quark content!

Example: π and ρ has the same quark content but $m_\pi \sim 140\,MeV$ and $m_\rho \sim 770\,MeV$

Solution: different masses can be attributed to spin-spin interaction

In QED:

- potential energy between two magnetic dipoles has a term proportional to the scalar product of the two dipole moments

For Dirac particles with masses m_i, m_j:

$$U \propto \frac{e}{m_i}\vec{S}_i \cdot \frac{e}{m_j}\vec{S}_j \propto \frac{\alpha}{m_i m_j}\,\vec{S}_i \cdot \vec{S}_j \Rightarrow \text{weak in QED } \alpha \sim 1/137$$

In QCD $\alpha_s \sim 1$ (at low energy)

$$U \propto \frac{\alpha_s}{m_i m_j}\,\vec{S}_i \cdot \vec{S}_j \Rightarrow \text{„chromomagnetic" spin-spin interaction}$$

expectation value

$$m(q_1 q_2) = m_{q_1} + m_{q_2} + \frac{A}{m_1 m_2}\langle \vec{S}_1 \cdot \vec{S}_2 \rangle$$

$$\vec{S}^2 = \vec{S}_1^{\,2} + \vec{S}_2^{\,2} + 2\vec{S}_1 \cdot \vec{S}_2 \Rightarrow \vec{S}_1 \cdot \vec{S}_2 = \frac{\vec{S}^2 - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}}{2} \Rightarrow$$

$$\langle \vec{S}_1 \cdot \vec{S}_2 \rangle = \frac{1}{2}\left[\langle S^2\rangle - \langle S_1^2\rangle - \langle S_2^2\rangle\right] = \frac{1}{2}\left(s(s+1) - s_1(s_1+1) - s_2(s_1+1)\right)$$

$s_1 = s_2 = \frac{1}{2}$ and s is the total spin of the system

For pseudoscalar mesons $s=0$, for vector mesons $s=1 \Rightarrow$

Pseudoscalar mesons $(s=0)$: $m_p = m_1 + m_2 - \frac{3A}{4m_1m_2}$

Vector mesons $(s=1)$: $m_v = m_1 + m_2 + \frac{A}{4m_1m_2}$

$\Rightarrow m_p < m_v$

The formulae above describe the meson masses well for
$m_d = m_u = 0{,}307\,\text{GeV}$, $m_s = 0{,}49\,\text{GeV}$, $A = 0{,}06\,\text{GeV}^3$

except for the η' which is predicted to be $m_{\eta'} = 355$ but was found to be $m_{\eta'}^{exp} \sim 958\,\text{MeV}$. The reason for the discrepancy is attributed to η' being a flavourless singlet state that can mix with possible purely gluonic bound states

Baryon masses:

Same argument as for mesons:

$$m(q_1q_2q_3) = m_1 + m_2 + m_3 + A'\left(\frac{\langle \vec{S}_1\cdot\vec{S}_2\rangle}{m_1m_2} + \frac{\langle \vec{S}_2\cdot\vec{S}_3\rangle}{m_2m_3} + \frac{\langle \vec{S}_1\cdot\vec{S}_3\rangle}{m_1m_3}\right)$$

Good agreement for $m_d = m_u = 0{,}365\,\text{GeV}$, $m_s = 0{,}54\,\text{GeV}$, $A' = 0{,}026\,\text{GeV}^3$

Higher „masses" of the quarks found for mesons and much larger than the „free-quark" masses. These are „effective" masses (QCD potential)